Study of Steady State and Unsteady State Heat Transfer to Viscous Fluids

PHILLIP M. ROOKE and FRANKLIN J. STERMOLE

Department of Chemical and Petroleum-Refining Engineering
Colorado School of Mines, Golden, Colorado 80401

In this study of heat transfer to a viscous liquid in laminar flow, emphasis is placed on determining temperature profiles in the heated liquid during conditions of steady and unsteady flow conditions. The radial dependence of the dynamic behavior of point temperatures has been investigated for a viscous, high Prandtl number fluid. The specific system is the constant-flux heating of a viscous liquid in an upflow in a vertical circular tube. Temperatures were measured at three radial positions at the end of the heated region during conditions of steady flow and following step and sinusoidal flow rate disturbances.

Although the transfer of heat to liquids in laminar flow has received considerable attention, experimental studies have generally been concerned with measuring "mixing-cup" temperatures and correlating the findings as Nusselt numbers (*1-4*). As pure laminar flow is difficult to obtain for nonisothermal conditions, experimental data describing radial temperature profiles for nonisothermal conditions are scarce. Charm (*5*), however, has presented determinations of center-line temperatures in heated pseudoplastic fluids and Griskey and Wiehe (*6*) have developed experimental techniques which allow accurate determination of radial temperature variations in flowing polymers. The data for heated, flowing polymers would, however, seem to be of limited applicability because of the extremely high viscosities involved.

The dynamic temperature behavior of liquids in laminar flow has received essentially no experimental treatment, but the problem has been the object of considerable analytical study (*7-11*). Most of these studies have been concerned with temperature and heat flux upsets and are consequently of no particular interest here, but two investigations (*11, 12*) have considered the effects of flow rate disturbances. The solutions, however, are of an extremely complicated form, and their complexity probably hinders an understanding of the problem.

The experimental data gathered in this study were used to check an existing mathematical description (*13*) of the steady state temperature distribution. The complex dynamic solutions existing in the literature were not considered, but rather a simplified model was derived to aid in analyzing the results.

EXPERIMENTAL PROCEDURE

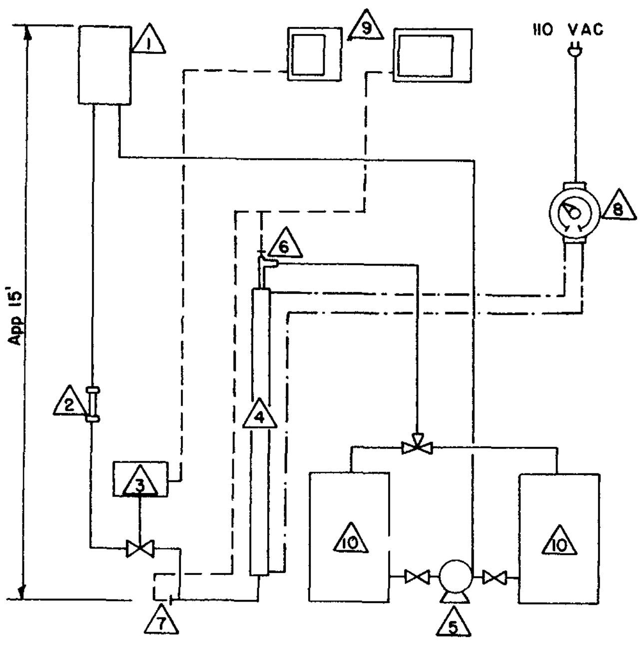
Obtaining reliable data for the heating of liquids in laminar flow involves balancing various conflicting factors. It is necessary to achieve fully developed thermal conditions at the end of heating, and this means that if the heated region is to be of reasonable length, the Reynolds number must be kept extremely low. A low Reynolds number, on the other hand, promotes natural convection in the liquid which disrupts the flow fields and consequently causes noise in the recorded data. The principal problem is thus to achieve true streamline flow at a very low Reynolds number. The experimental equipment and operating conditions used in this study reflect a compromise between these factors.

Good data were obtained by operating at a very low Reynolds number and heating with a low flux. The low Reynolds number was produced by a low flow rate in combination with a relatively high viscosity. The mild heating conditions of course dictated that electrical heating be used because this method is the most easily controlled.

The experimental apparatus is diagrammed in Figure 1. A glycerol-water solution contained in one of the tanks was pumped up to the constant-head tank and allowed to flow by gravity through a control valve and then upward through the heated test section. At the exit of the heated section, the fluid temperature was measured at three radial positions and recorded. After leaving the heated zone, the solution passed into the other holding tank.

The heated test section was made from a 9-ft.-long piece of 1/2-in. nominal steel pipe (I.D. 0.622 in.). The last 8 ft. of the tube were wrapped with 20-gauge Nichrome wire at a wrap rate of 4/in. The heating coil was then covered with a layer of asbestos cloth and 1.5 in. of commercial pipe insulation to reduce heat losses. The length immediately preceding the heated zone served as an hydrodynamic calming section to assure the complete development of velocity profiles at the beginning of heating. The end terminals of the heating wire were connected to a 15-amp. variable transformer which stepped down a 110-v., a.c. electrical source.

The flow rate of the liquid was controlled by a 0.25-in. plug valve fitted with a 21-in. long handle. The handle of the valve was connected to a linear potentiometer so that valve



1 Constant head tank
2 Filter
3 Sine wave generator
4 Heated test section
5 Pump
6 Thermocouple holder
7 Thermocouple cold junction
8 Variable transformer
9 Recorders
10 Holding tanks

Fig. 1. Diagram of experimental apparatus.

Correspondence concerning this article should be addressed to Professor F. Stermole.

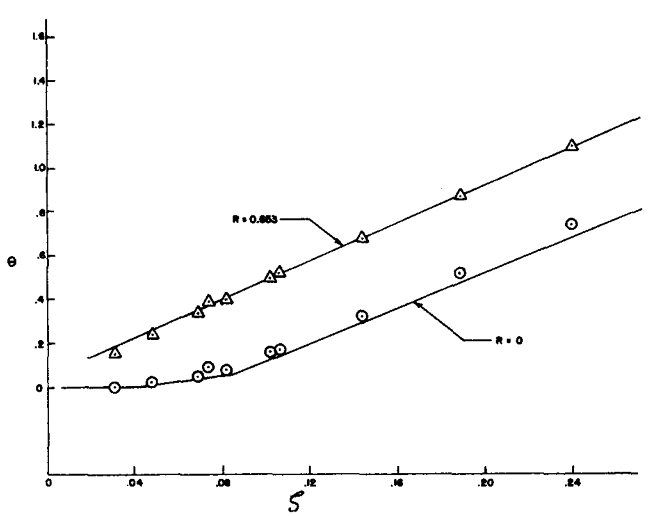


Fig. 2. Steady state outlet temperatures for N_{Re} between 5 and 37; legend: ○, △ experimental; —— theoretial, Equation (1).

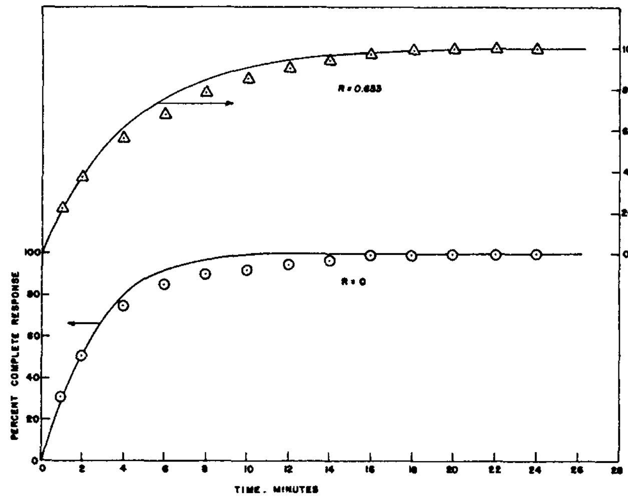


Fig. 3*b*. Temperature response to step decrease in flow rate from $N_{Re} = 11$ to $N_{Re} = 5$; see Figure 3*a*.

position, and hence flow rate, could be monitored. A 1.5-volt battery provided an electrical source for the recording of the valve position.

Sinusoidal flow rate variations were produced by connecting the valve handle to a mechanical sine-wave generator. The generator consisted of a cam driven by an electric motor whose speed was reduced through three variable speed drives connected in series. The variable speed drives effectively reduced the speed of the motor so that very low frequencies of approximately 1 cycle/hr. to 1 cycle/min. could be obtained quickly and precisely.

Temperatures of the heated fluid were measured by supporting three thermocouples parallel to the axis of the tube at dimensionless positions of $R = 0$ and $R = \pm\ 0.653$. The thermocouples were iron-constantan junctions shielded in 1/16-in.-diam. stainless-steel tubes. Cold junctions were located at the entrance of the heated section so that the readings were measurements of the difference between inlet and outlet temperatures. The thermocouple signals were recorded on a Hewlett-Packard model 7100B strip chart recorder. During sinusoidal flow rate upsets, the recorder receiving thermocouple signals was synchronized with the recorder monitoring valve position so that phase lags could be determined.

The experimental work described in this paper was performed using a solution of 75 wt. % glycerol and 25 wt. % water ($N_{Pr} = 262$ at 68°F.). Similar experiments using other liquids are described in reference *14* with comparable results.

The apparatus was operated to obtain temperature profiles during steady and time-variant flow conditions. Step response data were gathered in conjunction with frequency-response data, so that the magnitude of the step changes was equal to the amplitude of the sinusoidal upset. The recorded data were of very good quality, being free of noise during both steady and time-variant operation.

The experimental steady state data presented in Figure 2 in dimensionless-variable form represent temperature increases obtained at operation of runs for flow rates between Reynolds numbers of 5 and 37. The heat flux used here and in all experimental runs was 1.47 B.t.u./sq.ft./min. The two temperatures obtained at $R = 0.653$ were averaged to give a single temperature because of the symmetry of the temperature distribution and agreement of the temperatures within ± 0.4°F. The data show that fully developed conditions were encountered for some runs as well as incompletely developed conditions for other runs at high flow rates because the center-line temperature increase decreases to zero at the higher flow rates.

Transient response to step increases and decreases in flow have been reduced to percent complete response for comparison with mathematical descriptions. Figures 3*a* and 3*b* contain the results of a step flow rate increase and a step decrease between Reynolds numbers of 5 and 11 ($\zeta = 0.240$ and $\zeta = 0.102$); Figures 4*a* and 4*b* contain the results of step changes between Reynolds numbers of 11 and 24 ($\zeta = 0.102$ and $\zeta = 0.048$). Response to a sinusoidal flow rate disturbance of a constant amplitude equal to the step changes described in Figures 3*a* and 3*b* is shown in Figure 5.

At the very low Reynolds numbers encountered, natural convective effects would perhaps be expected to present a problem. In order to assess the extent of the distortion of the flow field

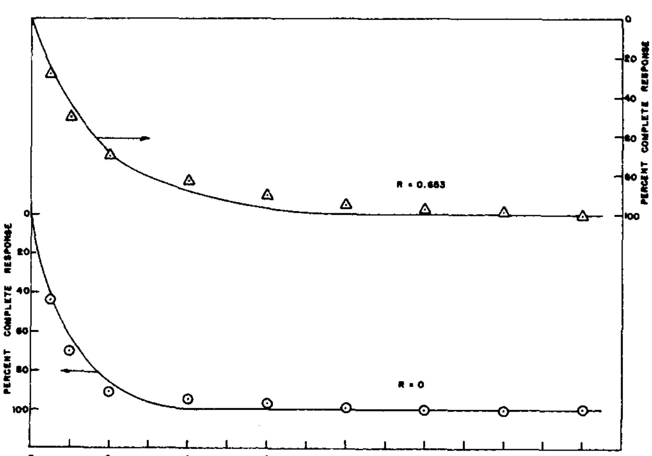


Fig. 3*a*. Temperature response to step increase in flow rate from $N_{Re} = 5$ to $N_{Re} = 11$; legend as in Fig. 2. See Equation (6).

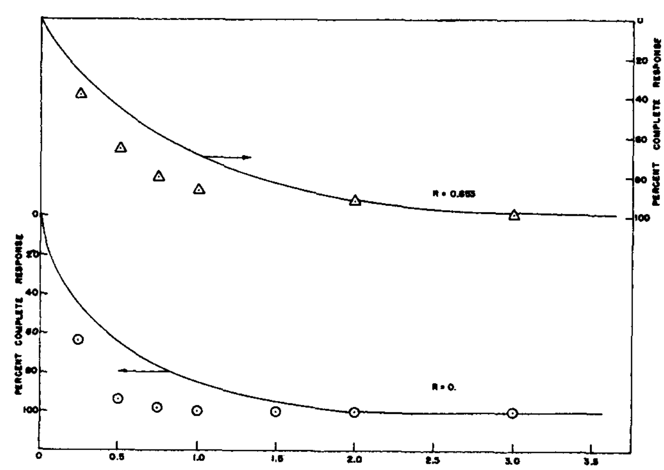


Fig. 4*a*. Temperature response to a step increase in flow rate from $N_{Re} = 11$ to $N_{Re} = 24$; see Figure 3*a*.

due to heating, the experimental conditions were compared with the data of Scheele and Hanratty (*15*) who described the extent of flow-field distortion in terms of the magnitude of the N_{Gr}/N_{Re} ratio. Their data suggest that the radial velocity profile ceases to be parabolic and experiences an inversion at the center at $N_{Gr}/N_{Re} = 32.94$. For the worst conditions of the present work (that is, the lowest Reynolds number), $N_{Gr}/N_{Re} = 16.4$ at the end of heating indicating that perhaps some distortion is present. Also, the temperature-dependent nature of the fluid viscosity could be expected to contribute a small amount of flow field distortion.

The nonlinear nature of the physical system was obvious in the frequency response data, for the response was not always

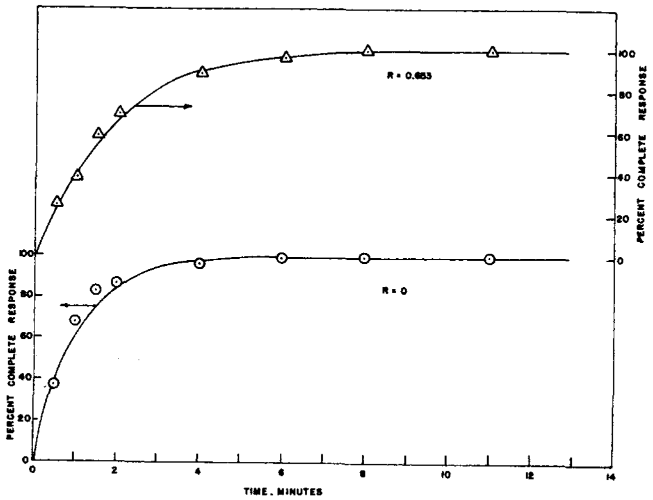


Fig. 4b. Temperature response to a step decrease in flow rate from $N_{Re} = 24$ to $N_{Re} = 11$; see Figure 3a.

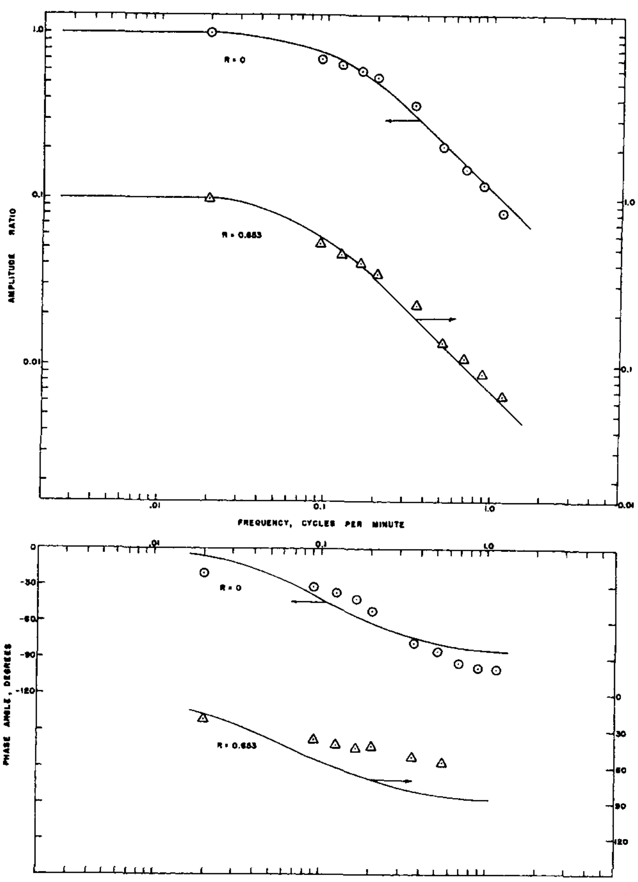


Fig. 5. Frequency response results for the same range of upsets and with the same legend presented in Figure 3.

sinusoidal. That the response is not necessarily a sine wave can be visualized by considering the response of the temperature at $R = 0$. If the amplitude of the flow rate upset is large enough to cause the entrance region to extend beyond the zone of heating, the temperature at $R = 0$ will decrease to the entrance temperature and give no response. Thus, for such a sinusoidal upset, the output would resemble a "flattened" sine wave.

Because of the nonlinear nature of the problem, standard frequency response methods of analysis are not applicable. The temperature response to sinusoidal flow rate upsets was therefore considered to demonstrate the recovery characteristics of the temperatures.

The dependence of temperature responses on radial position is obvious in the graphs shown in Figures 3*a*, 3*b*, 4*a*, 4*b*, and 5. The center-line temperature responds faster to step increases and step decreases than does the outlying temperature. Less attenuation during response to a sinusoidal upset is also apparent for the center-line temperature. These differences are due to the velocity distribution across the tube as the faster velocity at $R = 0$ gives this position the ability to respond more quickly to a disturbance.

MATHEMATICAL DESCRIPTIONS

Prediction of temperature increases in steady laminar flows subjected to constant flux heating is complicated by the presence of the thermal entrance region. If it is assumed, as suggested by Eckert (*16*), that fully developed conditions exist beyond the entrance length, then temperature profiles in this region may be described by the solution given by Bird, Stewart, and Lightfoot (*13*)

$$\theta = -4\zeta - R^2 + \frac{1}{4}R^4 + \frac{7}{24} \tag{1}$$

where

$$\theta = \frac{T - T_0}{\dfrac{q r_w}{k}} \qquad R = \frac{r}{r_w} \qquad \zeta = \frac{zk}{\rho C_p V_{\max} r_w^2}$$

The steady state solution was obtained by simplifying the governing partial differential equation with the assumption that temperature increases in the axial direction in the region of fully developed thermal conditions are linear. An approximate, lumped-parameter description of the dynamic behavior can be obtained in a similar manner if fully developed conditions are assumed. The temperature increase can then be represented as

$$\tau = Gz + f(r) \tag{2}$$

where τ is the temperature increase $T - T_0$.

The gradient G and the radial temperature dependence $f(r)$ are both functions of time, but the time dependent characteristics of $f(r)$ can be ignored. This assumption will allow $f(r)$ to be represented by its steady state description given in Equation (1)

$$f(r) = \frac{q r_w}{k}\left(\frac{r^2}{r_w^2} - \frac{1}{4}\frac{r^4}{r_w^4}\right) \tag{3}$$

The time-dependent energy equation

$$V\frac{\partial \tau}{\partial z} + \frac{\partial \tau}{\partial t} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(\frac{r\partial \tau}{\partial r}\right) \tag{4}$$

can now be simplified by substitution of Equations (2) and (3) through appropriate differentiation. These operations reduce equation (4) to

$$VG + z\frac{dG}{dt} = \frac{4\alpha q}{k r_w}\left(1 - \frac{r^2}{r_w^2}\right) \tag{5}$$

The specific disturbances producing dynamic responses to be considered here are the step and sinusoidal flow rate upsets. A normalized solution of Equation (5) for step upsets in flow is

$$\text{percent complete response} = 100\left(1 - e^{-\frac{ut}{z}}\right) \quad (6)$$

where U is the velocity following the upset.

Nonlinearities are introduced into Equation (5) when sinusoidal flow rate upsets are involved and consequently linearization techniques produce the easiest closed-form solution. A linearized solution by Laplace transform methods yields the following amplitude ratio and phase angle relations:

$$AR = \frac{G_s}{z} \quad \frac{1}{\frac{V_s}{z} + \omega^2} \quad (7)$$

$$\phi = -\tan^{-1}\left(\frac{z\omega}{V_s}\right) \quad (8)$$

The s subscripts denote a value about which a perturbation occurs.

This simplified first-order description of the dynamic behavior is dependent only on the length, velocity, and frequency. The model would be expected to predict the transient responses quite accurately since the total response would be essentially complete after one residence time, regardless of the higher order effects. During frequency responses, however, the model could be expected only to indicate trends of the amplitude ratio and phase angle because under these circumstances higher order effects are usually significant.

Solutions of Equations (1), (6), (7), and (8) are compared with corresponding experimental findings in Figures 2, 3*a*, 3*b*, 4*a*, 4*b*, and 5.

DISCUSSION

The comparison in Figure 2 demonstrates the approximate validity of Equation (1) in providing a good description of the temperature distribution in a steady state laminar flow system subjected to constant-flux heating. The small difference between experimental and predicted values at $R = 0$ can perhaps be explained as the effects of natural convection. These effects would tend to distort the velocity distribution in the direction of plug flow and hence increase center-line temperatures.

Bird, Stewart, and Lightfoot (*13*) point out that Equation (1) was derived by realizing "... that the constant heat flux through the tube wall will result in a rise in the fluid temperature that is linear in ζ. One further expects that the shape of the radial temperature profiles will ultimately not undergo further change with increasing ζ." It is interesting to note that the degree of separation of the temperatures of the two radial points considered remains essentially the same in the regions of fully developed thermal conditions indicating that the assumption of a constant radial dependence is valid.

In predicting responses to step changes in flow rate, the simple model given in Equation (5) assumes that the response is based solely on length and velocity and consequently that any radial difference in response time is due only to velocity variations across the tube. Comparison of model predictions and experimental data in Figures 3*a*, 3*b*, 4*a*, and 4*b* indicate that the physical system is more complicated than this. Agreement between experimental and theoretical results is better for the dynamic response data at lower than higher Reynolds numbers because neglecting the entrance region length is a better assumption at low Reynolds numbers. Figure 2 shows that entrance region length is very significant as N_{Re} approaches 37 so it is not surprising that the model does not agree exactly with experimental data in this flow region.

Since a complete frequency-response analysis is not applicable to nonlinear systems such as the one considered here, the approximate model was expected to give only an estimate of dynamic trends. Figure 5 demonstrates that a reasonable qualitative prediction of the temperature response is obtained through the use of the lumped-parameter model described by Equations (7) and (8).

NOTATION

C_p = specific heat, B.t.u./lb.
f = radial temperature dependence, °F.
G = axial temperature gradient, °F./ft.
k = thermal conductivity, B.t.u./ft./hr., °F.
N_{Pr} = Prandtl number
N_{Re} = Reynolds number
q = local heat flux, B.t.u./sq.ft./hr.
r = radial coordinate
r_w = radius of tube, ft.
R = reduced radius, r/r_w
t = time, min.
T = temperature, °F.
U = velocity following upset in flow rate, ft./min.
V = velocity, ft./min.
z = axial coordinate

Greek Letters

α = thermal diffusivity, sq.ft./hr.
ζ = dimensionless residence time, $zk/\rho C_p V_{max} r_w^2$
θ = dimensionless temperature increase, $T-T_0/qr_w/k$
ρ = density, lb./cu.ft.
τ = temperature increase, °F.
ω = frequency, cycles/min.

Subscripts

0 = inlet condition
s = steady state value
w = tube wall conditions
max = maximum

LITERATURE CITED

1. Drew, T. B., J. J. Hogan, and W. H. McAdams, *Ind. Eng. Chem.*, **23**, 936-945 (1931).
2. Kirkbride, C. G., and W. L. Mccabe, *ibid.*, 625-631.
3. Sieder, E. N., and G. E. Tate, *ibid.*, **28**, 1429-1433 (1936).
4. Bergman, P. D., and L. B. Koppel, *AIChE J.*, **12**, 648-655 (1966).
5. Charm, S. E., *Ind. Eng. Chem.*, **1**, 79-82 (1962).
6. Griskey, R. G., and I. A. Wiehe, *AIChE J.*, **12**, 308-312 (1966).
7. Siegel, Robert, *J. Appl. Mech.*, **80**, 140-142 (1959).
8. Siegel, R., and E. M. Sparrow, *J. Heat Transfer*, **81**, 29-36 (1959).
9. Siegel, Robert, *J. Appl. Mech.*, **27**, 241-249 (1960).
10. Perlmutter, Morris, and Robert Siegel, *J. Heat Transfer*, **83**, 432-440 (1961).
11. Siegel, R., and M. Perlmutter, *ibid.*, **85**, 358-365 (1963).
12. Zang, H. K., "Heat Exchanger," D.Sc. thesis, Colorado School of Mines, Golden (1967).
13. Bird, R. B., W. E. Stewart, and E. N. Lightfoot, "Transport Phenomena," p. 293-297, 363-364, Wiley, New York (1960).
14. Rooke, P. M., D.Sc. thesis, Colorado School of Mines, Golden (1969).
15. Scheele, G. F., and T. J. Hanratty, *J. Fluid Mech.*, **14**, 244-256 (1962).
16. Eckert, E. R. G., and Drake, R. M., "Heat and Mass Transfer," p. 190-200, McGraw-Hill, New York (1959).

Manuscript received November 11, 1969; revision received June 11, 1970; paper accepted June 15, 1970.